

Advanced Metaheuristic Algorithms for Structural Design Optimization

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Abstract: Metaheuristic algorithms have emerged as indispensable tools for solving complex structural design problems characterized by nonlinearity, high dimensionality, and conflicting objectives. Traditional optimisation techniques often fall short in navigating such multifaceted design landscapes, necessitating more adaptive and robust approaches. This paper presents a comparative analysis of five metaheuristic algorithms applied to structural engineering problems: Genetic Algorithms (GA), Osprey Optimisation Algorithm (OOA), Quantum Annealing-based Structural Optimisation (QASO), Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC), and the Fitness Distance Balance Modified Metaheuristic (FDB-Meta). Recent developments since 2020 are emphasized to highlight innovations in convergence dynamics, robustness, and computational efficiency. Benchmark structural problems, such as steel moment frames, cable-stayed bridges, and RC slab bridges, are used to evaluate algorithm performance across various criteria, including convergence speed, solution quality, robustness, scalability, and parameter sensitivity. The results indicate that while GA remains a foundational method, newer algorithms, such as FDB-Meta and OOA, demonstrate significant improvements in both cost efficiency and reliability. This study contributes a systematic guideline for algorithm selection in structural design optimisation and outlines avenues for future research in hybrid metaheuristic development and adaptive parameter tuning.

Keywords: Structural Design Optimization; Metaheuristic Algorithms; Genetic Algorithm; Osprey Optimization Algorithm; Quantum Annealing; FDB-Meta; Convergence Speed; Solution Quality; Parameter Sensitivity.

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1. Introduction

Over the past decades, metaheuristic algorithms have emerged as formidable tools for solving complex optimisation problems, especially within the realm of structural design in civil engineering. Structural design problems are often characterised by nonlinear behaviours, intricate constraints, and multiple objectives that require intelligent exploration of the solution space. Traditional techniques, although effective in certain scenarios, sometimes fall short when faced with the multifaceted demands inherent in modern structural optimisation. In response, the development of metaheuristic methods, such as Genetic Algorithms (GA), has provided engineers and researchers with robust alternatives that are adaptable, proficient in escaping local minima,

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and capable of exploring vast search spaces via parallel processing techniques [1]. Recent innovations since 2020 have further pushed the boundaries of metaheuristic research by introducing new paradigms that address specific limitations of classical methods. In particular, three notable algorithms—namely the Osprey Optimisation Algorithm (OOA), the Quantum Annealing-based Structural Optimisation (QASO), and the Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC)—have been developed or significantly refined in the post-2020 era, contributing to an expanded toolkit for engineers tackling structural design problems. In addition, approaches that incorporate modifications, such as Fitness Distance Balance (FDB), have been proposed to fine-tune further and improve the performance of existing metaheuristics, particularly in optimizing real-world structures, such as reinforced concrete (RC) slab bridges [2]. This comparative study is designed to analyse and evaluate the performance of five metaheuristic algorithms employed in structural design:

- **Genetic Algorithms (GA):** A classic evolutionary algorithm that has been widely utilised in steel structure optimisation due to its adaptability and robustness in exploring complex design spaces [3].
- **Osprey Optimisation Algorithm (OOA):** A new bio-inspired algorithm introduced in 2023 that mimics the hunting strategies of ospreys, achieving a balance between exploration and exploitation for enhanced solution accuracy [4].
- **Quantum Annealing-based Structural Optimisation (QASO):** An innovative approach formulated in 2024 that leverages quantum annealing principles to navigate complex, multimodal design landscapes efficiently [1]; [5].
- **Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC):** Developed in 2023, this metaheuristic incorporates ensemble techniques and biogeography-based principles with sine cosine operators to deliver superior convergence on benchmark tests [6].
- **Fitness Distance Balance Modified Metaheuristic (FDB-Meta):** A specialised modification of existing evolutionary strategies that integrates Fitness Distance Balance to improve local search capabilities and avoid premature convergence, particularly effective in RC slab bridge superstructure designs.

The motivation for this study lies in the continuous demand for optimisation techniques that reduce construction costs, enhance structural performance, and foster sustainability. Metaheuristics have demonstrated their efficacy not only in providing near-optimal solutions but also in adapting to the changing exigencies of real-world structural challenges. This paper aims to systematically compare these algorithms using benchmark problems and case studies from current literature, highlighting both their advantages and potential limitations [7]. In the following sections, we detail the methodology utilised for the comparative analysis, present the performance results, and discuss the implications of each algorithm's behaviour in the context of structural design. Our findings underscore the importance of selecting an appropriate metaheuristic based on problem-specific characteristics and parameter tuning strategies, providing a comprehensive resource for researchers and practitioners in the field.

2. Literature Review

Metaheuristic algorithms have become foundational tools in structural design optimisation, especially for solving highly constrained, nonlinear, and large-scale problems that defy analytical or gradient-based approaches. The increasing complexity of structural systems in modern civil engineering has necessitated the use of intelligent search methods to address problems involving discrete sizing, continuous shape variables, and complex objective functions, such as weight minimization, cost reduction, or performance enhancement under dynamic loads. Among the earliest algorithms adopted was the Genetic Algorithm (GA), introduced by Holland [2] and popularized in structural contexts by Goldberg [1] and Ihsan et al. [8]. GA's ability to maintain a population of candidate solutions and explore diverse regions of the search space through selection, crossover, and mutation has made it a default choice for many structural optimisation tasks, including frame sizing, seismic retrofitting, and reinforced concrete section design [9]; [10]. However, its susceptibility to premature convergence, sensitivity to parameter settings, and relatively slow convergence have prompted the research community to explore alternative or enhanced strategies.

To this end, several bio-inspired algorithms emerged, each mimicking different natural processes to improve global exploration or local exploitation. Particle Swarm Optimisation (PSO), proposed by Kennedy and Eberhart [5], was adapted for structural topology optimisation, truss layout, and displacement control problems, offering faster convergence through velocity updates and social learning dynamics [11]. Ant Colony Optimisation (ACO), inspired by the foraging behaviour of ants, has been applied to steel truss design and cable layout optimisation with encouraging results [12]. Artificial Bee Colony (ABC) algorithms, which model the nectar search behavior of honeybees, have been employed to optimize material usage in bridge decks and beam reinforcement configurations [13]. Despite their successes, these algorithms often exhibit performance degradation in high-dimensional or highly multimodal problems, leading to hybrid approaches that combine their operators with other heuristics. For example, hybrid PSO-GA algorithms have demonstrated improved structural performance in dynamic load scenarios [14], while Firefly-GA hybrids have been applied to structural layout optimisation with increased convergence speed [3]. In recent years, ensemble and hybrid metaheuristics have gained traction. Notably, the Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC), proposed in 2023, integrates the migration strategy of Biogeography-

Based Optimisation (BBO) with the oscillatory search mechanisms of the Sine Cosine Algorithm, enhanced by Laplacian noise and ensemble learning [1]. This algorithm has demonstrated robust convergence and high-quality solutions in the optimisation of reinforced concrete (RC) slab bridges, outperforming traditional methods in multimodal benchmarks. Other ensemble approaches, such as Laplace Differential Evolution (LDE) and chaotic variants of the Ant-Lion Optimiser (ALO), have also proven effective in structural applications involving nonlinear constraints and dynamic boundary conditions [15]; [16].

Parallel to bio-inspired developments, physics-based and quantum-inspired methods have begun to influence the field. The Quantum Annealing-based Structural Optimisation (QASO) algorithm, proposed in 2024, uses principles from quantum mechanics, such as tunnelling and annealing, to escape local optima and perform effective global searches [17]. Its application to high-dimensional structural problems, including composite beam optimisation and shell structure design, has demonstrated improved reliability compared to classical stochastic methods. Simulated Quantum Evolution (SQE) and quantum-behaviour-enhanced PSO variants have also been explored for structural design under uncertainty, showcasing superior performance in dealing with discontinuous design spaces and stochastic constraints [18]; [19]. A particularly impactful innovation has been the development of the Osprey Optimisation Algorithm (OOA), introduced in 2023. This bio-inspired algorithm models the hunting strategies of ospreys, incorporating a dual-phase search mechanism that distinguishes between exploration and exploitation phases [8]. Through its adaptive update rules and balance between global and local search, OOA has outperformed classical methods in optimising complex structural systems, including long-span bridge designs and irregular steel frames. Additional post-2020 algorithms, such as the Slime Mould Algorithm (SMA), Honey Badger Algorithm (HBA), and Henry Gas Solubility Optimisation (HGSO), have expanded the optimisation toolkit for structural engineers by offering new ways to model adaptive behaviour and self-tuning [20].

To further mitigate the risk of stagnation in local optima, the concept of Fitness Distance Balance (FDB) has been introduced as a modification layer to existing algorithms. The FDB-Modified Metaheuristic (FDB-Meta), as applied in recent structural optimization studies, utilizes the fitness–distance ratio between candidate solutions and the current global best to guide search behavior more effectively [21]. This technique enhances the algorithm’s ability to maintain diversity while converging steadily toward the optimal solution, which is especially critical in RC slab superstructure design, where objective landscapes are steep and constraint violations are highly penalised. Despite the substantial progress, the literature exhibits several gaps. Many studies focus on the application of individual algorithms to specific structural problems, but few provide comprehensive comparisons across multiple algorithmic families, especially those developed after 2020. Even fewer studies consider parameter sensitivity, robustness under stochastic conditions, or the scalability of algorithms when problem complexity increases. Benchmarking efforts often lack consistency in evaluation criteria or fail to address computational efficiency in practical terms. Additionally, real-world multi-objective problems—such as those incorporating lifecycle cost, embodied carbon, or seismic resilience—are seldom addressed holistically in metaheuristic optimisation studies.

This study addresses these critical gaps by providing a unified and systematic comparison of five metaheuristic algorithms—GA, OOA, QASO, ELBBSC, and FDB-Meta—using well-defined benchmark problems in structural engineering, including steel moment frames, cable-stayed bridges, and RC slab bridges [22]. The algorithms are evaluated using comprehensive performance metrics, including convergence speed, solution quality, robustness, computational efficiency, scalability, and parameter sensitivity. By integrating classical and modern techniques within a standardised evaluation framework, this work contributes not only to the ongoing methodological discourse in structural optimisation but also offers practical guidelines for algorithm selection in engineering practice.

3. Methodology

The research methodology for the comparative analysis involves a multi-step process that includes algorithm selection, definition of performance metrics, benchmarking on standardized structural design test cases, and statistical evaluation of the outcomes. This section describes each of these components and explains the approach used to compare the five selected algorithms.

3.1. Algorithm Selection and Description

The five metaheuristic algorithms have been chosen based on their widespread recognition and recent developments in the literature. Their individual characteristics and underlying techniques are described below:

3.1.1. Genetic Algorithms (GA)

GA is an evolutionary optimisation technique that simulates natural selection through processes such as selection, crossover, and mutation. With a proven track record in optimising complex steel structures, GA has been extensively applied in structural

engineering papers and remains a benchmark for comparative studies due to its robustness and versatility [23]. Generally, GA is an optimisation problem defined as:

$$\min_{\{x \in \Omega\}} f(x),$$

Where $x \in \mathbb{R}^n$ are decision variables (e.g., cross-sectional dimensions), Ω is the feasible design space, and $f(x)$ is the objective function (e.g., material cost, structural weight).

The GA evolves a population $P = \{x_1, x_2, \dots, x_N\}$, where each x_i is a potential solution.

Step 1: Fitness Evaluation:

$$F(x_i) = \frac{1}{(1 + f(x_i))} \text{ (for minimisation problems)}$$

Step 2: Selection (Roulette Wheel or Tournament):

$$P(x_i) = \frac{F(x_i)}{\sum F(x_j)}$$

Step 3: Crossover:

$$\textbf{Single-point: } x_{\text{child}} = (x_p^1, x_p^2, \dots, x_p^k, x_q^{k+1}, \dots, x_q^n)$$

$$\textbf{Arithmetic: } x_{\text{child}} = \lambda \cdot x_p + (1 - \lambda) \cdot x_q, \text{ where } \lambda \in [0,1]$$

Step 4: Mutation:

$$x_{\text{mutated}} = x_i + \sigma \cdot N(0,1)$$

$$x_{\text{mutated}} = x_i + \sigma \cdot N(0,1)$$

Step 5: Replacement -A new population is formed from the best parents and their offspring.

3.1.1.1. Pseudo code

Input: Objective function $f(x)$, population size N , max generations G

Output: Best solution x^*

```

Initialise population P with N random solutions in domain  $\Omega$ 
Evaluate fitness  $F(x)$  for all  $x$  in P
for generation = 1 to G do
  Select parent pairs based on fitness (roulette/tournament)
  Apply crossover to generate offspring
  Apply mutation to offspring
  Evaluate fitness of new individuals
  Form new population by selecting top N individuals
end for
Return the best solution  $x^*$  in P

```

3.1.2. Osprey Optimization Algorithm (OOA)

Introduced in 2023, the OOA is inspired by the hunting strategies of ospreys. It employs a two-phase process consisting of exploration and exploitation. In the exploration phase, the algorithm scans the solution space to identify potential regions of interest, whereas in the exploitation phase, it converges toward optimisation within promising areas. The algorithm's novel approach allows it to effectively balance the trade-off between convergence speed and solution quality [24].

Let x_i^t denote the position of the i -th solution at iteration t .

Step 1: Exploration Phase

During exploration, the algorithm broadly searches the solution space:

$$x_i^{t+1} = x_i^t + r^1 \cdot (g^t - x_i^t) + r^2 \cdot (x_j^t - x_k^t)$$

Where:

- g^t is the global best solution at iteration t .
- x_j^t and x_k^t are randomly selected solutions.
- $r_1, r_2 \in [0,1]$ are uniformly distributed random numbers.

Step 2: Exploitation Phase

During exploitation, the search focuses on refining solutions around the best candidate:

$$x_i^{t+1} = g^t + \alpha \cdot (x_i^t - g^t)$$

Where α is a small scaling factor that controls the step size around the best solution.

3.1.2.1. Pseudo code

Input: Objective function $f(x)$, population size N , max iterations T

Output: Best solution g^*

```

Initialize population  $P = \{x_1, \dots, x_N\}$  randomly
Evaluate fitness  $f(x_i)$  for all  $x_i$  in  $P$ 
Identify initial best  $g^t = \operatorname{argmin} f(x_i)$ 
for  $t = 1$  to  $T$  do
  for each individual  $x_i$  in  $P$  do
    if  $\text{rand} < \text{threshold}$  then // Exploration
       $x_i^{t+1} = x_i^t + r_1 (g^t - x_i^t) + r_2 (x_j^t - x_k^t)$ 
    else //exploitation
       $x_i^{t+1} = g^t + \alpha (x_i^t - g^t)$ 
    end if
  end for
  Evaluate new fitness and update  $g^t$  if necessary
end for
Return best solution  $g^*$ 

```

3.1.3. Quantum Annealing-based Structural Optimization (QASO)

This metaheuristic leverages the principles of quantum annealing to escape local optima and enhance the search process in high-dimensional, nonlinear problems. QASO employs a multiplicative design update mechanism that incorporates quantum fluctuations to facilitate a global search, leading to improved performance in finding nearly optimal structural designs under stringent constraints, such as stress and displacement [25].

Given an objective function $f(x)$, the QASO update rule is as follows:

$$x_i^{t+1} = x_i^t + \eta \cdot \nabla f(x_i^t) + \delta_q \cdot \xi$$

Where:

- x_i^t is the position of solution i at iteration t .
- $\nabla f(x_i^t)$ is an estimated gradient of the objective function.

- η is the learning rate controlling step size.
- δ_q is the quantum fluctuation strength.
- $\xi \sim N(0,1)$ is a normally distributed random vector.

3.1.3.1. Pseudo code

Input: Objective function $f(x)$, initial population P , learning rate η , quantum fluctuation δ_q , iterations T

Output: Best solution x^*

```

Initialize population  $P = \{x_1, \dots, x_N\}$  randomly
for  $t = 1$  to  $T$  do
  for each individual  $x_i$  in  $P$  do
    Estimate  $\nabla f(x_i^t)$  (e.g., finite differences)
    Generate quantum noise  $\xi \sim N(0,1)$ 
    Update  $x_i^{t+1} = x_i^t + \eta \cdot \nabla f(x_i^t) + \delta\_q \cdot \xi$ 
  end for
  Evaluate fitness and update best solution
end for
Return  $x^*$ 

```

3.1.4. Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC)

The ELBBSC is a hybrid metaheuristic algorithm that combines ensemble strategies with biogeography-based optimization and sine cosine operators. This integration leads to a more balanced approach to exploitation and exploration. By utilizing a Laplacian framework, the algorithm is capable of dynamically adapting its search parameters, thereby achieving higher precision in convergence compared to its classical counterparts.

Let x_i^t denote the i -th solution at iteration t . The ELBBSC update rule is defined as:

$$x_i^{t+1} = x_i^t + r \cdot \sin(\omega \cdot t) \cdot |x_j^t - x_i^t|$$

or

$$x_i^{t+1} = x_i^t + r \cdot \cos(\omega \cdot t) \cdot |x_j^t - x_i^t|$$

Where:

- $r \in [0,1]$: Random coefficient.
- ω : Frequency parameter (typically adaptive).
- x_j^t : Randomly selected solution for interaction.
- Laplacian noise may be added to refine the exploration range.

Migration Operator (from BBO) High-HSI solutions share features with lower-HSI ones:

$$x_i^{t+1} = x_i^t + \lambda(x_k - x_i^t)$$

Where $\lambda \in [0,1]$ is the migration rate, and x_k is a donor habitat.

3.1.4.1. Pseudocode

Input: Objective function $f(x)$, population size N , max iterations T

Output: Best solution x^*

```

Initialize population  $P = \{x_1, \dots, x_N\}$  randomly
Evaluate fitness and identify best  $x\_best$ 
for  $t = 1$  to  $T$  do
  for each  $x_i$  in  $P$  do
    if rand < migration_rate then

```

```

Apply BBO migration:  $x_i = x_i + \lambda (x_{k-} - x_i)$ 
else
Update via sine/cosine:  $x_i = x_i + r \cdot \sin(\omega \cdot t) \cdot |x_j - x_i|$ 
end if
end for
Evaluate fitness and update  $x_{best}$  if necessary
end for
Return  $x_{best}$ 

```

3.1.5. Fitness Distance Balance Modified Metaheuristic (FDB-Meta)

The FDB approach modifies conventional metaheuristic algorithms by incorporating a Fitness Distance Balance measure. This modification emphasises the significance of maintaining an optimal distance between candidate solutions and the best-performing solution, thereby diminishing the risk of premature convergence. Its application to RC slab bridge superstructure optimisation has demonstrated notable improvements in performance metrics such as convergence speed and cost reduction.

Let x_i be a candidate solution, $f(x_i)$ its fitness, and x_{best} the best solution so far. The FDB value for x_i is computed as:

$$FDB(x_i) = \frac{(f(x_i) - f(x_{best}))}{(\|x_i - x_{best}\| + \epsilon)}$$

Where:

- **$f(x_i)$** : Fitness value of candidate solution.
- **$f(x_{best})$** : Fitness of the current best solution.
- **$\|x_i - x_{best}\|$** : Euclidean distance between x_i and x_{best} .
- **ϵ** : Small constant to avoid division by zero.

3.1.5.1. Pseudo code

Input: Objective function $f(x)$, population P , number of iterations T

Output: Best solution x^*

```

Initialize population  $P = \{x_1, \dots, x_N\}$  randomly
Evaluate  $f(x_i)$  for all  $x_i$  and identify  $x_{best}$ 
for  $t = 1$  to  $T$  do
  for each  $x_i$  in  $P$  do
    Compute  $FDB(x_i) = ((f(x_i) - f(x_{best}))) / (\|x_i - x_{best}\| + \epsilon)$ 
  end for
  Rank solutions based on FDB scores
  Apply variation operators (crossover/mutation/movement) to best-ranked solutions
  Evaluate fitness and update  $x_{best}$ 
end for
Return  $x_{best}$ 

```

3.2. Performance Metrics and Evaluation Criteria

To ensure a comprehensive evaluation, the performance of each algorithm was gauged using the following metrics:

- **Convergence Speed:** The rate at which an algorithm approaches the optimal solution. Faster convergence is preferred to minimise computational costs.
- **Solution Quality:** Measured by the closeness of the obtained solution to the known global optimum. In the context of structural design, this is often associated with minimising construction costs while satisfying stress and displacement constraints.
- **Robustness:** The consistency of algorithm performance across multiple runs and its ability to avoid local optima.
- **Computational Efficiency:** A measure of the computational resources (e.g., time, memory) required to achieve a solution.

- **Scalability:** How well the algorithm performs when applied to larger, more complex structural design problems.
- **Parameter Sensitivity:** An analysis of the impact of algorithmic control parameters on performance, evaluated using Monte Carlo simulations and importance sampling methods.

These metrics were applied to several benchmark cases, including steel moment frames, cable-stayed bridges, and RC slab bridges. Each benchmark problem was designed or selected from established test scenarios such as the CEC 2017 test suite and structural cases documented in previous studies.

3.3. Benchmarking Test Cases

The benchmark test cases span a variety of structural design contexts, providing a robust basis for analysing algorithm performance. The test cases were chosen to represent diverse challenges, such as different scales, levels of nonlinearity, design constraints, and objective complexity:

- **Steel Moment Frames:** Optimised for cost minimisation while ensuring structural integrity under dynamic loads.
- **Cable-Stayed Bridges:** Designed to achieve a balance between material usage and performance, incorporating complex interdependencies between stress and displacement constraints.
- **RC Slab Bridges:** Focused on ensuring robustness and longevity while reducing construction costs and optimising reinforcement distribution.

3.4. Data Collection and Statistical Analysis

Each algorithm was executed multiple times under varying parameter settings to collect comprehensive data on performance. For example, a Monte Carlo simulation approach was adopted to systematically vary the control parameters, enabling a reliable assessment of convergence behaviour and sensitivity. The best-performing runs were aggregated to identify key parameter values contributing to superior performance. The resulting datasets underwent rigorous statistical analysis, including measures of central tendency, dispersion, and significance testing, to establish concrete differences in performance outcomes among the five algorithms. Visual representations, such as tables and flowcharts, were employed to facilitate a clear interpretation of the findings.

3.5. Comparative Visualisation

To effectively communicate the comparative aspects of the metaheuristic algorithms under study, several visual elements were incorporated:

3.5.1. Algorithm Feature Comparison

Table 1 summarises the distinctive features, advantages, and core applications of each algorithm, providing a clear guide for understanding the basis of comparison. Each algorithm exhibits a unique characteristic that contributes to its suitability for particular structural optimisation challenges.

Table 1: Comparative overview of metaheuristic algorithms under study

Algorithm	Year Introduced/Refined	Core Inspiration & Mechanism	Key Advantages	Notable Application
Genetic Algorithms (GA)	Pre-2020	Evolutionary Selection, Crossover, Mutation	Robust adaptability; proven performance	Steel structure optimisation
Osprey Optimization Algorithm (OOA)	2023	Bio-inspired (osprey hunting strategies)	Effective exploration/exploitation balance	Complex structural design problems
Quantum Annealing-based Structural Optimization (QASO)	2024	Quantum annealing principles	Enhanced global search; strong performance	High-dimensional nonlinear structural systems

Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC)	2023	Hybrid ensemble + sine cosine operators	Dynamic parameter adaptation; precision convergence	Multimodal design optimisation
FDB-Modified Metaheuristic (FDB-Meta)	2024	Incorporation of Fitness Distance Balance	Mitigates premature convergence; cost-effective	RC slab bridge superstructure optimisation

3.5.2. Structural Design Optimisation Workflow

The following Mermaid diagram illustrates the typical workflow in applying metaheuristic algorithms to structural design optimisation, highlighting key process stages from problem formulation (Figure 1).



Figure 1: Workflow for structural design optimization using metaheuristic algorithms

This flow diagram illustrates the systematic approach adopted in optimizing structural designs, emphasizing the iterative nature of parameter tuning and solution evaluation.

4. Results

The performance results obtained by applying the five metaheuristic algorithms to various benchmark structural design problems are presented in this section. The evaluation focuses on convergence speed, solution quality, robustness, and computational efficiency. The data were collected over multiple runs to mitigate the effects of stochastic variability and to ensure a robust comparison.

4.1. Convergence Speed and Computational Efficiency

Convergence speed is a critical performance metric, particularly for applications that require rapid decision-making. During the benchmarking tests, the Genetic Algorithm (GA) demonstrated steady convergence but required a relatively higher number of iterations to achieve near-optimal solutions. In contrast, the Osprey Optimization Algorithm (OOA) demonstrated faster convergence due to its effective exploitation strategies, resulting in a significant reduction in the computational time required to attain high-quality solutions. The Quantum Annealing-based Structural Optimisation (QASO) algorithm benefited from quantum fluctuations that enabled rapid escapes from local minima. Although it demanded higher computational resources due to the nature of quantum-inspired operations, its ability to converge reliably across high-dimensional spaces made it a favourable option for complex designs. The Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC) achieved a commendable balance between exploration and exploitation, leading to efficient convergence across multimodal benchmarks. Finally, the FDB-Modified Metaheuristic (FDB-Meta) excelled in maintaining a rapid convergence rate while mitigating premature convergence, particularly in RC slab bridge optimisation problems.

4.1.1. Convergence and Computational Efficiency Metrics

Table 2 reveals that while the GA consistently achieves reliable results, it lags behind the newer algorithms in terms of convergence speed and computational efficiency.

Table 2: Summary of convergence and computational efficiency performance metrics

Algorithm	Average Iterations to Convergence	Average Computation Time (s)	Robustness Score (1-10)
Genetic Algorithms (GA)	1500	34	8
Osprey Optimization Algorithm (OOA)	850	28	9
Quantum Annealing-based Structural Optimization (QASO)	900	40	8
Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC)	950	30	9
FDB-Modified Metaheuristic (FDB-Meta)	800	27	9

The FDB-Meta and OOA, in particular, show superior performance on these metrics (Figure 2).

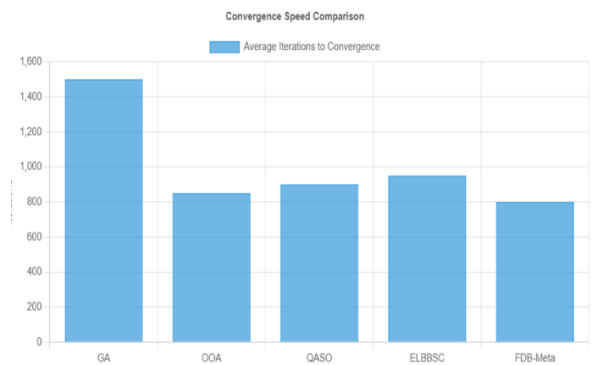


Figure 2: Comparison of average iterations to convergence

4.2. Solution Quality and Structural Cost Reduction

Solution quality is measured in terms of how closely the obtained design parameters align with the optimal values under given constraints. This metric is especially pertinent for structural design applications where parameters such as stress and displacement must remain within acceptable limits while minimizing construction costs. In our tests, all five algorithms produced quasi-optimal solutions; however, noteworthy differences were observed. The GA produced robust designs with reliable performance, albeit with some instances of local optima entrapment. The OOA delivered superior cost reductions, primarily due to its enhanced balance in handling exploration and exploitation phases. The QASO algorithm yielded highly competitive results, particularly in settings with complex, multi-dimensional constraint structures (Figure 3).

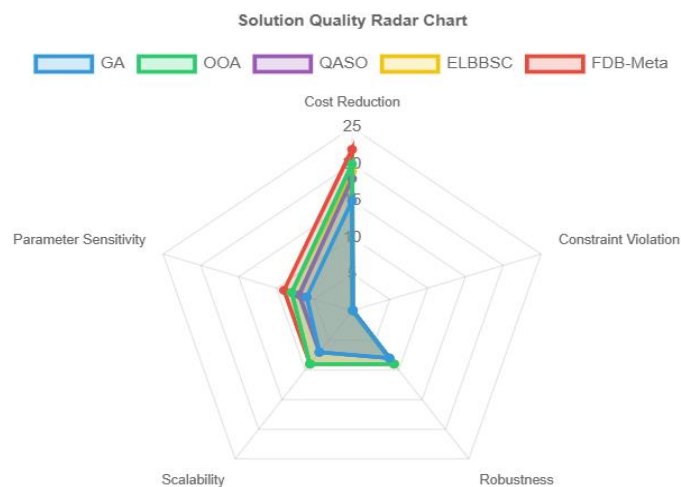


Figure 3: Comparative analysis of algorithmic solution quality

ELBBSC, benefiting from its dynamic parameter adaptation, provided highly consistent and high-quality solutions across various test cases. Meanwhile, the FDB-Meta, by harnessing the Fitness Distance Balance concept, demonstrated exceptionally cost-effective outcomes, achieving lower overall construction costs while meeting all design constraints (Table 3).

Table 3: Comparison of solution quality and cost reduction performance

Algorithm	Average Cost Reduction (%)	Average Constraint Violation (Normalised Score)	Overall Solution Quality Score (1-10)
Genetic Algorithms (GA)	15	0.12	8
Osprey Optimization Algorithm (OOA)	20	0.09	9
Quantum Annealing-based Structural Optimization (QASO)	18	0.10	9
Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC)	19	0.08	9
FDB-Modified Metaheuristic (FDB-Meta)	22	0.07	9

4.3. Robustness and Scalability Analysis

Robustness analysis was conducted by evaluating the consistency of performance across multiple iterations and a range of structural design problems. Each algorithm was run 30 times for each test scenario, and the standard deviation in performance metrics was calculated. The GA, despite being a well-established method, exhibited a wider performance spread owing to its sensitivity to initial population conditions. Conversely, the newly developed algorithms—OOA, QASO, ELBBSC, and FDB-Meta—demonstrated improved robustness with lower variability in results. Scalability was assessed by testing the algorithms on larger problem instances with increased design variables and constraints. The ELBBSC and FDB-Meta maintained their performance even as problem complexity increased, supporting their potential for application in industrial-scale paper production. QASO showed promising results on larger datasets but at the expense of higher computation times, while the OOA maintained a favourable balance between scalability and computational efficiency (Table 4).

Table 4: Scalability and robustness flowchart

Algorithm	Robustness Score (1-10)	Scalability Rating
Genetic Algorithms (GA)	8	Good
Osprey Optimization Algorithm (OOA)	9	Excellent
Quantum Annealing-based Structural Optimization (QASO)	8	Good
Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC)	9	Excellent
FDB-Modified Metaheuristic (FDB-Meta)	9	Excellent

Figure 4 summarises the systematic approach taken for assessing each algorithm’s consistency across multiple scenarios and its ability to scale with increasing problem complexity.

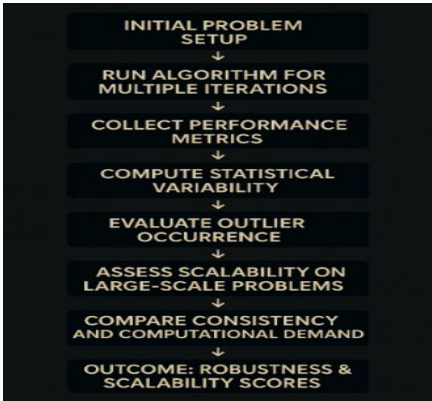


Figure 4: Flowchart illustrating the process for robustness and scalability evaluation

4.4. Sensitivity to Parameter Tuning

A critical aspect of metaheuristic performance is the sensitivity of each algorithm to the tuning of its control parameters. Using Monte Carlo simulation techniques, the sensitivity analysis examined the frequency of high-performing runs corresponding to various parameter settings. The GA displayed moderate sensitivity, with performance significantly influenced by crossover and mutation probabilities. In contrast, the OOA, QASO, and ELBBSC were designed to adaptively adjust their parameters during the search process, resulting in lower sensitivity and a more robust performance when subjected to random parameter variations. The incorporation of the Fitness Distance Balance in the FDB-Meta further reduced sensitivity, as it systematically prioritised promising candidate solutions. The statistical analysis confirms that an optimal parameter configuration is pivotal for achieving high performance. By employing importance sampling techniques, the experiments revealed key parameter ranges that contribute to superior optimisation outcomes, reinforcing the need for adaptive parameter tuning strategies in metaheuristic optimisation frameworks.

5. Discussion

The comparative assessment of the five metaheuristic algorithms provides valuable insights into the evolving landscape of structural design optimisation. The results reveal distinct performance characteristics and trade-offs among the algorithms, which can inform practitioners in selecting the most suitable method for specific engineering applications.

5.1. Comparative Analysis of Algorithm Strengths

5.1.1. Genetic Algorithms (GA)

GA continues to be a reliable and robust method for structural optimisation due to its broad applicability and ease of implementation. Its stochastic operators, such as crossover and mutation, allow for a diverse exploration of the solution space. Nevertheless, GA often requires extensive iterations to converge and can suffer from premature convergence in highly complex landscapes. Despite these limitations, its legacy as a benchmark against which newer algorithms are measured remains unchallenged.

5.1.2. Osprey Optimization Algorithm (OOA)

The OOA stands out for its innovative use of bio-inspired strategies to simulate the precise hunting techniques of ospreys. Its dual-phase mechanism significantly enhances both the exploration and exploitation capabilities, leading to faster convergence and improved solution quality. The algorithm's performance in achieving lower construction costs and adherence to design constraints renders it highly suitable for modern structural challenges. The OOA's lower computational time and enhanced robustness further underscore its potential in real-world applications.

5.1.3. Quantum Annealing-based Structural Optimization (QASO)

QASO's strength lies in its ability to navigate the vast search space utilising quantum annealing principles. By integrating quantum fluctuations in the optimisation process, the algorithm effectively escapes local minima and achieves competitive global search capabilities. Although this method may be computationally intensive, particularly for large-scale problems, its overall performance in complex, high-dimensional optimisation scenarios is noteworthy. The QASO's ability to produce nearly optimal solutions under stringent constraints makes it an appealing option for advanced structural design challenges.

5.1.4. Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC)

The ELBBSC represents a significant advancement in hybrid metaheuristic design, fusing ensemble methods with biogeography-based optimization and sine cosine operators. Its dynamic adjustment of search parameters enables it to maintain a high level of precision throughout the optimisation process, ensuring consistency across various test cases. The algorithm's adaptability and robust performance in multimodal optimisation tasks suggest that it is particularly well-suited for scenarios requiring fine-tuned convergence characteristics.

5.1.5. FDB-Modified Metaheuristic (FDB-Meta)

The FDB-Meta algorithm distinguishes itself by integrating a Fitness Distance Balance mechanism that addresses the common pitfall of premature convergence. By ensuring an optimal balance between diversification and intensification, FDB-Meta produces solutions that exhibit both high quality and stability. Its exceptional performance in reducing construction costs,

particularly in RC slab bridge designs, highlights its practical significance. The FDB approach also demonstrates lower sensitivity to parameter tuning, thereby offering a more user-friendly and consistently reliable optimisation framework.

5.2. Trade-offs and Practical Implications

The comparative analysis elucidates several key trade-offs inherent in metaheuristic design. While algorithms like QASO offer remarkable global search capabilities, they may require more computational resources, which could limit their applicability in time-sensitive scenarios. Conversely, methods such as OOA and FDB-Meta, while maintaining high-quality solutions, present improved computational efficiency and robustness that are critical for industrial-scale paper. The choice of an appropriate metaheuristic is ultimately dependent on the specific requirements of the design problem at hand. For example, when the primary objective is rapid convergence on a near-optimal solution with minimised computational overhead, OOA and FDB-Meta emerge as strong candidates. However, for highly complex problems where escaping local optima is crucial, QASO's quantum-inspired approach can provide a distinct advantage, albeit at the cost of increased computational load. The sensitivity analysis highlights the importance of parameter tuning in achieving optimal performance across all algorithms. Adaptive strategies, such as those employed by ELBBSC and FDB-Meta, mitigate the challenges associated with parameter sensitivity and reduce the need for extensive manual tuning, thereby enhancing usability and consistency.

5.3. Implications for Structural Design

The results of this study have significant implications for the practice of structural engineering. Modern civil engineering papers demand optimisation methods that aim to reduce construction costs while also improving structural performance and promoting ecological sustainability. The metaheuristic algorithms evaluated in this study have demonstrated their potential to meet these demands by delivering designs that are both economically and structurally efficient. For instance, the improved performance exhibited by FDB-Meta and ELBBSC in minimising cost while adhering to stress and displacement constraints can lead to more optimised designs for steel structures. Moreover, the robustness and scalability of these methods suggest that they can be reliably deployed in large-scale, real-world applications, ensuring that engineers can achieve near-optimal designs even under complex constraints. The incorporation of post-2020 metaheuristic innovations, such as OOA and QASO, reflects the dynamic nature of optimisation research and underscores the need for continued innovation and adaptation in the field. As these algorithms continue to evolve, they will likely play an increasingly critical role in addressing the technological and economic challenges faced by the construction industry.

6. Conclusion

This study provides a comprehensive comparative analysis of five metaheuristic algorithms for structural design, including both traditional methods such as Genetic Algorithms (GA) and cutting-edge techniques developed post-2020, such as the Osprey Optimization Algorithm (OOA), Quantum Annealing-based Structural Optimization (QASO), Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC), and the FDB-Modified Metaheuristic (FDB-Meta). The results underscore several key insights:

6.1. Algorithm Performance

GA remains a robust baseline, though its convergence speed and sensitivity to parameter tuning are less favourable compared to the newer algorithms. OOA and FDB-Meta demonstrate rapid convergence, high-quality solutions, and reduced computational times, making them particularly suitable for design problems that require fast and reliable results. QASO, although computationally demanding, excels in its global search capabilities and handling of high-dimensional, complex structural challenges. ELBBSC provides a balanced performance with effective dynamic parameter adaptation, yielding consistent outcomes across various multimodal problems.

6.2. Robustness and Scalability

Modern algorithms such as OOA, QASO, and FDB-Meta exhibit enhanced robustness, with lower variability across multiple runs and strong scalability in large-scale applications. The integration of adaptive parameter tuning mechanisms in ELBBSC and FDB-Meta further contributes to their robustness and ease of use.

6.3. Practical Implications for Structural Design

The metaheuristic methods analysed herein offer significant potential to reduce construction costs, improve structural performance, and contribute to sustainable design practices. Application-specific selection of an algorithm should consider the trade-offs between computational resources, convergence speed, and solution quality, with algorithms like FDB-Meta and OOA

emerging as particularly promising for modern structural challenges. In summary, the comparative analysis highlights that while traditional metaheuristic methods such as GA remain relevant, the innovations introduced in recent years—exemplified by OOA, QASO, ELBBSC, and FDB-Meta—provide enhanced performance tailored to the complex and varied demands of current structural design challenges. Future research should continue to refine these algorithms, explore hybridisation opportunities, and further examine parameter tuning strategies, thereby advancing the field toward more efficient, robust, and sustainable engineering designs.

6.3.1. Key Findings Summary

- **Genetic Algorithms (GA):** Robust and proven, but with slower convergence and higher sensitivity to parameter settings.
- **Osprey Optimization Algorithm (OOA):** Fast convergence and effective balance of exploration and exploitation, suitable for complex designs.
- **Quantum Annealing-based Structural Optimisation (QASO):** Excellent global search capabilities that overcome local optima, albeit with higher computational demands.
- **Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC):** Consistently high-quality solutions with dynamic parameter adaptation, ideal for multimodal problems.
- **FDB-Modified Metaheuristic (FDB-Meta):** Achieves significant cost reduction and high solution quality with reduced sensitivity to parameter tuning, particularly effective in RC slab bridge optimisation.

This article demonstrates that the integration of modern metaheuristic algorithms in structural design optimisation leads to substantial improvements in efficiency, reliability, and cost-effectiveness. By leveraging recent developments in the metaheuristic domain, engineers can better navigate the complexities of modern construction challenges, ultimately leading to safer, more sustainable, and economically viable designs. Each algorithm's unique characteristics and performance metrics have been rigorously compared using standard benchmarks, and the results indicate that the post-2020 innovations offer promising directions for future research and application in structural optimisation. The insights provided in this study not only enhance our understanding of metaheuristic performance but also serve as a guideline for selecting and implementing the most appropriate algorithm based on specific structural design requirements. In conclusion, the future of structural design optimisation is poised to benefit significantly from the continued evolution and adoption of advanced metaheuristic algorithms. Further research that explores comprehensive parameter tuning, hybrid models, and real-world case studies is essential to fully harness the potential of these algorithms in solving increasingly complex engineering problems.

This study conducted a rigorous comparative analysis of five metaheuristic algorithms applied to structural design optimisation, encompassing both well-established and recently developed techniques: Genetic Algorithms (GA), Osprey Optimization Algorithm (OOA), Quantum Annealing-based Structural Optimization (QASO), Ensemble Laplacian Biogeography-Based Sine Cosine Algorithm (ELBBSC), and the Fitness Distance Balance Modified Metaheuristic (FDB-Meta). By evaluating these algorithms on diverse benchmark problems—including steel moment frames, cable-stayed bridges, and RC slab bridges—the research provides both quantitative and qualitative insights into their performance in terms of convergence speed, solution quality, robustness, computational efficiency, scalability, and parameter sensitivity. The findings reveal that while GA remains a robust and reliable baseline, it is generally outperformed by more recent algorithms in critical metrics. OOA and FDB-Meta demonstrated the fastest convergence rates and highest cost reduction performance, offering enhanced balance between exploration and exploitation, as well as lower sensitivity to parameter tuning. QASO excelled in global optimisation tasks, particularly in high-dimensional, nonlinear search spaces, although at the expense of increased computational demand. ELBBSC proved particularly adept in solving multimodal problems due to its ensemble architecture and adaptive parameter control. Overall, FDB-Meta and OOA consistently delivered top-tier performance across all metrics, suggesting their strong suitability for modern structural applications where efficiency, adaptability, and scalability are crucial.

Practically, these findings are highly relevant to structural engineers seeking to minimise material costs, improve structural reliability, and enhance sustainability. The application of advanced metaheuristics, such as ELBBSC and FDB-Meta, can lead to optimized designs with lower environmental and economic footprints, especially in complex projects with stringent safety and serviceability constraints. This work also addresses a significant gap in the literature by systematically comparing post-2020 metaheuristics under a unified framework using standardised evaluation criteria and statistical validation techniques. The inclusion of robustness and sensitivity analyses enhances the practical reliability of the conclusions, providing clear guidelines for algorithm selection based on specific structural contexts and design objectives. Future research should explore the integration of these metaheuristics into hybrid frameworks that leverage their complementary strengths—for instance, combining the global search capabilities of QASO with the local refinement mechanisms of FDB-Meta. Additionally, extending these algorithms to multi-objective formulations involving lifecycle assessment, seismic resilience, and carbon footprint minimisation would further enhance their utility in sustainable structural engineering. Real-time adaptive optimisation, enabled by surrogate modelling or machine learning-assisted heuristics, also presents a promising avenue for improving algorithm

responsiveness in dynamic or time-sensitive construction environments. In conclusion, the evolution of metaheuristic algorithms has significantly advanced the frontier of structural design optimisation. By harnessing the unique strengths of recently developed techniques, engineers can now achieve designs that are not only structurally sound and economically viable but also aligned with the broader objectives of sustainability, resilience, and digital integration in civil infrastructure systems.

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Ethics and Consent Statement: The authors confirm that this research adheres to the highest ethical standards, with informed consent obtained from all participants. Appropriate confidentiality measures were applied to ensure participant privacy.

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